2024 Volume 2, Issue 2
Signal and Information Processing
ISSN: 2630-4805

Damage Identification of Frame Structures Based on Compressed Signal Processing

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Abstract:

In view of the huge amount of data generated in the process of structural health monitoring and the huge burden on data storage, a structural damage identification method based on compressed sensing is proposed. This method first reduces the dimension of the original signal by means of compressive perception theory and characterizes the original damage signal by replacing the original signal with the compressed signal. Then, the compressed signal is wavelet packet decomposition and damage identification is performed by constructing energy eigenvectors. In order to better reflect the advantages of compressive perception applied to structural damage identification, the processed compressed signal is compared with the original signal for damage identification. A three-story reinforced concrete frame is simulated by finite element software ABAQUS, which is verified by the damage identification method and numerical examples. The results show that the proposed method can accurately identify structural damage using compressed signals and obtain more accurate recognition results while reducing the computational load.

Keywords:

Compressed sensing
Damage identification
Wavelet packet decomposition
Characteristic energy vector
Numerical simulation

Online publication: December 16, 2024

1. Introduction

During its normal usage, civil engineering structures are subject to various influences from both internal and external sources, which can cause varying degrees of impact. As structural damage accumulates, it affects the safe use of the structure. Therefore, structural health monitoring is of great significance. However, the amount of data collected during structural health monitoring is quite large and cannot be directly applied.

Traditional signal acquisition methods involve uniform sampling based on the Nyquist sampling theorem and signal reconstruction through interpolation techniques. This requires a sampling frequency of at least twice the highest frequency of the signal, and often higher in practical applications, resulting in significant redundancy. Compressed sensing, on the other hand, has significant advantages in signal acquisition. It performs random sampling at a rate proportional to the sparsity of the signal, with a sampling frequency much lower than traditional methods. It simultaneously compresses the data during sampling, resulting in a smaller amount of observed data that contains the main information of the original data.

The theory of compressed sensing was formally proposed by Donoho *et al.* in 2006 and has been widely used in signal processing. Currently, researchers have introduced compressed sensing theory into structural health monitoring. Wang and Hao [1] were the first to introduce compressed sensing into structural damage identification in 2013. Li *et al.* [2] proposed an impedance data compression and reconstruction method that applies reconstructed data to structural damage identification using compressed sensing theory. Yao *et al.* [3] presented an iterative space-compressed sensing scheme for damage identification and localization, identifying damage from randomly collected sparse samples. Zheng and Yan [4] used compressed data instead of original data to characterize structural damage and quantitatively classify the severity of the damage. Chen *et al.* [5] obtained a small amount of data expressing characteristic information from the original data through compressed sensing theory and used the measured values for intelligent fault diagnosis.

In this paper, a damage identification method based on compressed signal processing (CSP) is proposed. The compressed signal is decomposed using wavelet packets for damage identification, avoiding the reconstruction of the original signal and reducing the burden of data storage and transmission.

2. Basic theory

2.1. Compressed sensing

Compressed sensing exploits the sparsity of signals in a certain transform domain, breaking through the limitations of traditional signal acquisition methods. It projects signals into a lower-dimensional space using linear random observations, compressing the signals and preserving damaged information during signal acquisition. This results in a small number of observations that contain the main information of the original signal, which can then be reconstructed using nonlinear optimization algorithms [6]. CSP is a research field of compressed sensing (CS) theory. It directly acquires signals in compressed form using CS, allowing for either the recovery of the original signal using CS reconstruction algorithms or post-processing directly on the compressed measured signals.

For a one-dimensional signal x of length N, there exists an N x N sparse basis Ψ, such that the signal x can be linearly expanded in the basis Ψ. The sparse representation of x is then given by:

$$x = \Psi\theta \tag{1}$$

In the formula, $\Psi = [\Psi_1, \Psi_2, \ldots, \Psi_N]$ coefficient vector $\theta = [\theta_1, \theta_2, \ldots, \theta_N]^T$.

If there are only a few non-zero elements in θ, then the signal x is a sparse signal under the sparse transformation basis Ψ. Under these conditions, projecting the signal x onto an M x N dimensional observation matrix Φ yields an M-dimensional observation signal y, where M is much smaller than N. The process of compressed measurement can be expressed as:

$$y = \Phi x \tag{2}$$

In the formula: y contains the main information of the original sparse signal x, and the number of elements in y is much smaller than that in x. By solving a linear system of equations, x can be reconstructed from y. However, since M is much smaller than N, the equation has infinitely many solutions, making it an underdetermined problem. Combining the formulas, y can be rewritten as:

$$y = \Phi x = \Phi\Psi\theta = \Theta\theta \tag{3}$$

In the formula, the observation matrix Φ and the signal's sparse space Ψ are uncorrelated. The sensing matrix Θ = ΦΨ indirectly reconstructs the original signal x through θ, reducing the number of unknowns in the linear system of equations and narrowing the solution space. The sparsity of the original signal affects the reconstruction quality, so the sensing matrix must satisfy the restricted isometry property (RIP) condition during signal reconstruction [7]. That is:

$$(1-k)\|\theta\|_2^2 \leq \|\Theta\theta\|_2^2 \leq (1+k)\|\theta\|_2^2 \tag{4}$$

In the formula, if the sensing matrix Θ satisfies the RIP condition with a constrained isometry constant k ∈ (0, 1), then the coefficient vector θ can be uniquely determined by solving the convex optimization problem under the l1 norm, achieving accurate reconstruction.

$$\begin{cases} min\ \|\theta\| \\ s.t.\ y = \Theta\theta \end{cases} \tag{5}$$

Compressed sensing mainly consists of sparse representation, design measurement matrix, and reconstruction algorithm. Based on the signal characteristics, this paper selects discrete cosine transform (DCT) for sparse representation of the signal, uses a random Gaussian measurement matrix for compressed sampling of the signal, and reconstructs the signal based on the orthogonal matching pursuit (OMP) algorithm from the greedy algorithms. The reconstruction error between the original signal and the reconstructed signal is represented using an error value, and the evaluation metric is expressed as:

$$\Delta = \left|\frac{\sum(\bar{x}-x)}{N}\right| \tag{6}$$

In the formula, $\bar{x}$ represents the reconstructed signal, x represents the original signal, and Δ represents the error value.

Although the traditional CS framework makes signal acquisition easier, it is not suitable for real-time monitoring due to its computationally expensive and power-consuming reconstruction process. In this context, the signal processing framework based on CSP, where compressed measurements carry sufficient information about the original signal, is more appropriate. Its inherent advantage is the omission of the reconstruction process, which also allows for the relaxation of constraints on the minimum number of compressed measurements imposed by signal recovery. This means that more undersampling can be performed without worrying about signal recovery.

2.2. Introduction to wavelet packet theory

Essentially, wavelet packets are a combination of a set of wavelet basis functions that exhibit both orthogonal properties and time-frequency characteristics. Wavelet packet analysis can simultaneously decompose both the low-frequency and high-frequency components of a signal [8]. Each layer has a different resolution, and every sub-band at each layer can occupy the full frequency range of the original signal, matching the signal spectrum and improving time-frequency resolution [9].

Performing an i-level wavelet packet decomposition on the signal x(t) yields $N = 2^i$ frequency bands, which can be represented as:

$$x_j^i(t) = \sum_{-}^{+} c_{j,k}^i(t) u_{j,k}^i(t) \tag{7}$$

In the formula, x_j^i represents the j-th wavelet packet at the i-th level, while $x_{j,k}^i(t)$, $u_{j,k}^i(t)$ denote the wavelet packet coefficients and functions, respectively.

The energy of a wavelet packet component can be expressed as:

$$E_j^i = \int_{-}^{+} [x_j^i(t)]^2\, dt \tag{8}$$

Due to the orthogonality of wavelet packets, the total energy E_i is equal to the sum of the energies in all signal components at the same scale, that is:

$$E_i = \int_{-}^{+} x^2(t)dt = \sum_{i=1}^{2i} E_j^i \tag{9}$$

2.3. Construction of energy feature vector

The energy characteristics of a signal are highly sensitive to changes in structural response, revealing some inherent features of the signal. By constructing a feature vector with energy as its elements, if the structure sustains damage, its frequency decreases, and the signal components obtained after decomposing the response signal may experience an increase or decrease in energy in certain components. The damage information of the structure is contained within these components. Wavelet packet decomposition allows the signal to be decomposed into arbitrarily fine frequency bands, enabling energy statistics to be performed on these bands to form a feature vector [10].

The energy of the signal within each frequency band at the i-th level is used to construct the feature vector T.

$$T = [E_{i,0}, E_{i,1}, \cdots, E_{i,N-1}] \tag{10}$$

Let $E = \sum_{i=1}^{2i} E_j^i$, and normalize it as follows:

$$T = [E_{i,0}/E, E_{i,1}/E, \cdots, E_{i,N-1}/E] \tag{11}$$

3. Numerical example

To verify the effectiveness of damage identification using compressed signals as described in this paper, a three-story reinforced concrete frame structure model was established using the finite element software ABAQUS. The specific parameters of the model are as follows: beam and column cross-sectional dimensions of 300 mm × 300 mm, materials C25 and Q345, Poisson's ratio μ of 0.3, elastic modulus E of 200 GPa, model span of 2800 mm, and story height of 2500 mm, as shown in **Figure 1**. For finite element analysis, Gaussian white noise was used as the excitation form, applied at the base of the model to excite the structure, with a duration of 20 seconds and a sampling frequency of 200 Hz. The acceleration response data of the structure was extracted from the finite element analysis results, and damage was simulated by reducing the elastic modulus of local components.

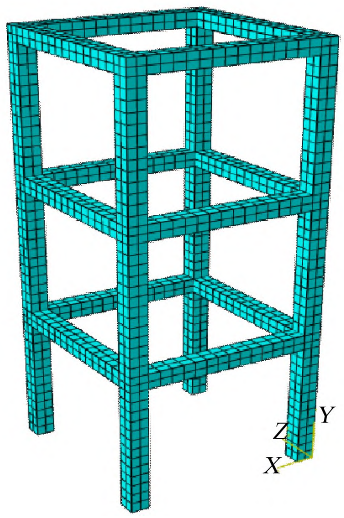

Figure 1. Schematic diagram of the structural model

The dynamic acceleration response signal of the top floor of the structure was obtained under white noise excitation, and the acceleration time history curve in the damaged state is shown in **Figure 2**. The changes in the first five natural frequencies of the structure in the damaged and undamaged states, obtained through structural dynamic analysis, are presented in **Table 1**. As shown in the table, there is no significant change in the frequencies before and after structural damage, with a maximum change of only 0.72%. This indicates that the sensitivity of natural frequency characteristics for structural damage identification is limited.

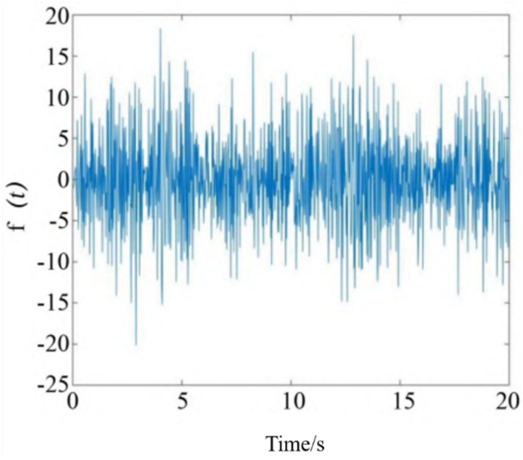

Figure 2. Acceleration of the damaged top floor

Structural damage identification is discussed using wavelet packet decomposition. The sampling frequency is 200 Hz, the analysis frequency is 100 Hz, the Dmey wavelet is chosen as the basis function, and the decomposition level is set to 5, resulting in 32 frequency bands with a bandwidth of 3.125 Hz each. To explore the impact of compressed response signals on identification results, two scenarios are established by comparing the identification results of uncompressed signals with those of compressed signals processed through compressed sensing. Scenario 1 involves a 30% reduction in the elastic modulus of local components on the top floor. Scenario 2 involves compressed sensing processing of the acceleration response obtained from Scenario 1, using wavelet packet energy as the feature principal component. The signal's energy is primarily focused on obtaining compressed signals for identification. The

Table 1. Changes in the first five natural frequencies of the structure

Frequency order	Undamaged	Damaged 30%	Frequency change/%
1	3.89	3.87	-0.51
2	6.88	6.86	-0.29
3	12.45	12.36	-0.72
4	15.32	15.25	-0.46
5	17.51	17.48	-0.17

Table 2. Frequency ranges corresponding to characteristic frequency bands

Node number	(5,1)	(5,2)	(5,4)	(5,3)	(5,7)	(5,8)	(5,6)	(5,5)
Frequency band range	[0~3.13]	[3.13~6.25]	[6.25~9.38]	[9.38~12.5]	[12.5~15.63]	[15.63~18.75]	[18.75~21.88]	[21.88~25]
Node number	**(5,13)**	**(5,14)**	**(5.16)**	**(5,15)**	**(5,11)**	**(5,12)**	**(5,10)**	**(5,9)**
Frequency band range	[25~28.13]	[28.13~31.25]	[31.25~34.38]	[34.38~37.5]	[37.5~40.63]	[40.63~43.75]	[43.75~46.88]	[46.88~50]

Table 3. Relative energy changes in characteristic frequency bands in working condition 1

Frequency band	No damage	30% damage	Energy change/%
(5,2)	4.466	5.574	+24.81
(5,4)	5.917	4.001	-32.38
(5,3)	20.653	10.296	-50.15
(5,7)	6.674	16.051	+140.5
(5,8)	11.795	8.536	-27.63

frequency bands corresponding to the first 16 nodes after wavelet packet decomposition are selected, specifically at nodes (5, 2), (5, 4), (5, 3), (5, 7), and (5, 8). Therefore, the energy values of the five wavelet packets in these frequency bands are chosen to form the feature vector, as shown in **Table 2**. Based on the frequency band ranges in **Table 2**, the first five natural frequencies of the structure are located at nodes (5, 2), (5, 4), (5, 3), (5, 7), and (5, 8). The changes in wavelet packet energy in characteristic frequency bands relative to the undamaged structure are presented in **Table 3**. When the structure is damaged, its frequency decreases. The energy at nodes (5, 2) and (5, 7) increases, while the energy at nodes (5, 4), (5, 3), and (5, 8) decreases. The relative energy distribution of the first 16 nodes is shown in **Figure 3**. To improve the robustness of the damage index, only the energy values from nodes with higher energy are typically selected.

Working condition 2 uses the response acceleration obtained from working condition 1 for compressed sensing processing, projecting it onto a Gaussian random matrix to obtain compressed signals, as shown in **Figure 4**. The sampling rate is 2, and the reconstruction algorithm selects the OMP algorithm. The calculated reconstruction error is 0.0353, indicating that accurate signal reconstruction can be achieved with good results. To verify whether the compressed acceleration response contains most of the damage information of the original signal, it is used for structural damage identification. The compressed undamaged and damaged signals are decomposed using wavelet packets, and the same processing is performed as in working condition 1. The relative energy distribution of the wavelet packet energy on the first 16 nodes is shown in **Figure 5**. The energy changes in the characteristic frequency bands after

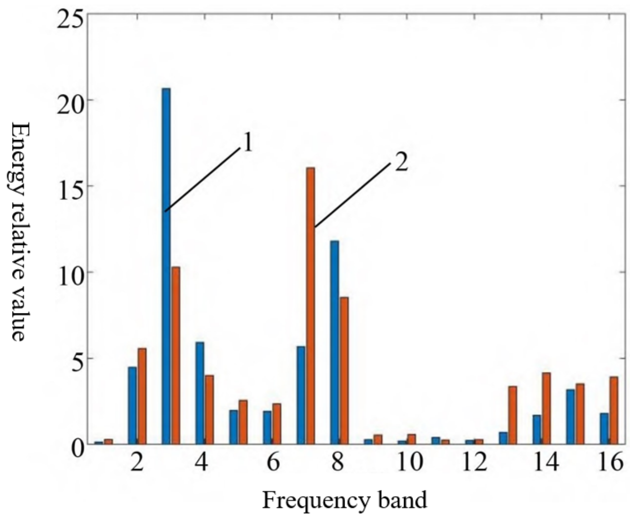

Figure 3. Distribution of wavelet packet energy in working condition 1 (1: No damage; 2: 30% damage)

Table 4. Relative energy changes in characteristic frequency bands after compression

Frequency band	No damage	30% damage	Energy change/%
(5,2)	6.782	9.742	+43.64
(5,4)	4.604	9.622	+108.99
(5,3)	26.613	20.312	-23.68
(5,7)	9.813	14.121	+43.90
(5,8)	15.067	12.255	-18.66

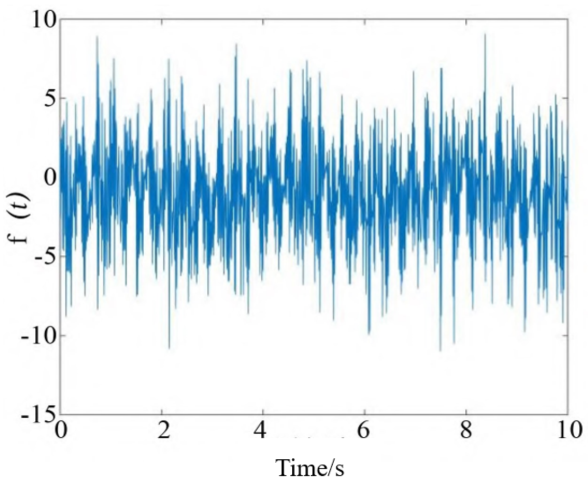

Figure 4. Compressed response acceleration

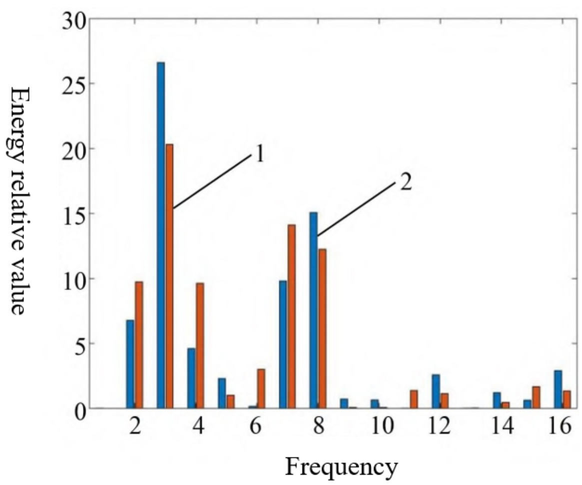

Figure 5. Distribution of wavelet packet energy in working condition 2 (1: No damage; 2: 30% damage)

wavelet packet decomposition relative to the undamaged structure are presented in **Table 4**. When the structure is damaged, its frequency decreases, and the energy on nodes (5, 2), (5, 4), and (5, 7) increases, while the energy on nodes (5, 3) and (5, 8) decreases. From **Tables 3** and **4**, it is clear that both working conditions can identify damage, indicating that the compressed signal contains most of the damage information of the original signal. The processed compressed signal can be used for structural damage identification and achieves relatively accurate identification results.

4. Conclusion

Addressing the issues of data storage, transmission, and computational burden associated with the massive data generated during structural health monitoring, this paper proposed an identification method that combines compressed sensing and wavelet packet analysis. The compressed signal processing method based on compressed sensing theory is applied to structural damage identification, and a finite element model of a frame structure is established and analyzed. Numerical simulation results showed that when performing structural damage identification, the changes in energy feature vectors obtained through wavelet packet decomposition are much greater than changes in signal frequency, indicating that energy feature vectors are more sensitive to structural damage. In this paper, compressed signals processed directly using compressed sensing are utilized for damage identification, demonstrating that compressed signals contain the main damage information of the original sparse signals. This approach not only reduces the amount of data but also ensures relatively accurate identification results. By comparing the effects of original signals and compressed signals on damage identification results, the superiority and feasibility of this method are verified, providing data support for the intelligent development of damage identification.

Disclosure statement

The authors declare no conflict of interest.

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