

The Definition of “Less than or Equal to” or “Less than and Equal to” Depending on the Context

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Abstract

In mathematics, the symbol “ \leq ” is often interpreted as “less than or equal to”. However, behind this simple definition lies a deeper duality, which can subtly change into “less than and equal to” in a different situation. It shows fascinating properties, especially when applied to mathematical intervals. In this article, we will explore the complexity of the symbol “ \leq ” in depth, analyze its different meaning in a variety of situations, and emphasize its unique role in standard inequality and interval mathematics. Furthermore, this deeper exploration will also strengthen our connection to the language of mathematic: when speakers say “less than or equal to” to express the symbol “ \leq ”, such expression is literally insufficient and inaccurate, and “not greater than” or other proper expressions may be used instead. By the same token the two different definitions of the symbol “ \geq ” can also be easily analyzed and concluded, and in similarity “not less than” or other proper expressions may be used to express the symbol “ \geq ” instead of the current “greater than or equal to”.

Keywords

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Inequality
Interval mathematics
Less than or equal to
Greater than or equal to
Different meaning

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1. To begin, let's review the basic definition of the symbol “ \leq ”

It indicates that one number is less than or equal to another number, and is one of the common symbols of comparison in mathematics. However, this simple relationship can be interpreted very differently in different fields and issues. Especially in the concept of mathematical intervals, the meaning of the symbol “ \leq ” becomes more colorful.

Taking into account mathematical intervals, we find that the symbol “ \leq ” does much more than represent numerical size relationships. In interval representation, it can be used to define a continuous range of values, including the values represented by the end points of the interval. In this case, we should think of it as a “less than and equal to” relationship, which exhibits that one interval can be a sum of less than the other interval and equal to the other interval to keep the same logical conformity

with its basic definition. For example, any positive whole number value in the semi closed interval $(-\infty, 5]$, including 1, 2, 3, 4 and 5 itself, satisfies the conditions of a positive whole number interval “less than” and “equal to” 5 at the same time, reflecting the different meaning of the symbol “ \leq ” in interval mathematics.

On further investigation, we find that in the standard inequality, the symbol “ \leq ” still expresses the traditional “less than or equal to” relation. In these situations, it emphasizes the relative relationship of numerical values, which is the basis for solving inequality problems. However, in interval mathematics, the context of the symbol “ \leq ” has undergone a subtle shift and has a broader meaning.

In summary, the symbol “ \leq ”, although usually interpreted as “less than or equal to,” exhibits the properties of “less than and equal to” in the context of mathematical intervals so as to keep the same logical conformity. This property gives the symbol “ \leq ” a different meaning and makes it play a unique role in interval mathematics. By deeply studying the complexity of this symbol, we can better understand its meaning in different mathematical fields, so that it can be more effectively applied to solve various problems^[1].

2. Revealing the subtle nature of the symbol “ \leq ”: context determines interpretation

In mathematics, the symbol “ \leq ” is a general comparison symbol used to represent a relationship between numbers or expressions. It has long been widely understood as an indicator of “less than or equal to.” However, there is a subtler nature behind this seemingly simple definition, and its interpretation is influenced by a particular context. This article will delve into dual interpretations of the symbol “ \leq ”, revealing its diversity in different fields and contexts. Especially in the field of mathematical intervals, the symbol “ \leq ” shows the characteristic of dual identity.

3. The traditional and present understanding of the symbol “ \leq ” as “less than or equal to” plays a dominant role in mathematics

It is the cornerstone of our exploration of the relationship

between the size of numerical values, and helps us to establish the basic framework of mathematical thinking. However, this traditional understanding is only the tip of the iceberg of the symbol “ \leq ”, which reveals a richer connotation and use in the deeper mathematical field. In the beginning of mathematics, we are often led to understand the relationship between the symbol “ \leq ” and “less than or equal to.” For example, when we encounter the inequality $a \leq b$, we intuitively interpret it as a being less than or equal to b . This understanding is essential for comparing the size relationships of numerical values. In numerical comparison, we can use this symbol to determine whether one value is less than or equal to another value, and thus deduce the order between the values.

This traditional understanding is useful in solving practical problems. For example, in finance, we can use the symbol “ \leq ” to determine whether the return of one investment is less than or equal to that of another investment, thereby making more informed decisions. In engineering, we can use this symbol to compare whether different materials are strong enough to meet specific requirements. However, when we delve deeper into the broader field of mathematics, the connotation presented by the symbol “ \leq ” becomes more complex and richer. In interval mathematics, this symbol not only represents the numerical size relationship but also emphasizes inclusiveness. Consider the interval $(-\infty, 10]$, which includes the one endpoint 10. In this context, the symbol “ \leq ” expresses not only “less than or equal to”, but also contains a special relationship: less than and equal to. This makes our knowledge expand from a single size relationship to the recognition of interval boundary values.

In summary, the traditional understanding of the symbol “ \leq ” as “less than or equal to” is fundamental to mathematical learning and helps us build a framework for numerical comparisons. However, in the deeper field of mathematics, it takes on a richer connotation, especially in interval mathematics, where the inclusive relationship is emphasized. Therefore, from the beginning of mathematics to the higher level of mathematical exploration, the symbol “ \leq ” plays an indispensable role in enriching our understanding of the mathematical world.

4. Dual identity of the symbol “ \leq ” in mathematical intervals

When we apply the symbol “ \leq ” to the concept of a mathematical interval, its meaning subtly changes. A mathematical interval is a continuous range of values, usually defined by one or two endpoints. In this context, we can think of the symbol “ \leq ” as the relation of “less than or equal to.” For example, for the closed interval $[0, 1]$, which contains 0 and 1 itself, the symbol “ \leq ” indicates not only the relation of greater than or equal to 0 and less than or equal to 1, but also the applicability to 0 and 1. This dual identity is of great significance in interval mathematics and provides a new perspective for us to understand the connotation of the interval ^[2].

5. Context-dependent interpretation of the symbol “ \leq ”

In short, the symbol “ \leq ” is often interpreted as “less than or equal to”, but its interpretation depends on the specific context. In traditional numerical comparison, it represents the relationship between the sizes of numerical values and is an indispensable tool in mathematics. However, the aforesaid interpretation is only applicable to the individual numbers, whereas in mathematical intervals, it has to exhibit the property of “less than and equal to” to demonstrate the full length of an interval of all contained individual numbers to keep the same logical conformity, which endows the symbol “ \leq ” the concept of dual identity. Therefore, when using the symbol “ \leq ,” we must carefully consider its specific field and context to ensure that we accurately convey the meaning we want to express.

6. Explore the dual meanings of the symbol “ \leq ” in depth

An in-depth study of the dual meanings of the symbol “ \leq ” can expand our understanding of mathematics. From the traditional numerical comparison to the dual identity in interval mathematics, the different interpretations of the symbol “ \leq ” in different situations enrich our understanding of the flexibility of mathematical symbols. By more detailed analysis of the application of the symbol “ \leq ” in various fields, we can better apply it to solve practical

problems and pursue a deeper understanding in the field of mathematics.

7. Interpretation of standard inequality and symbol “ \leq ”

Normally, we have a universal interpretation of the symbol “ \leq ” that is rooted in the standard concept of inequalities. This symbol is used to represent the relationship between two quantities. The educational platform “Cuemath” elaborates on this, noting that it combines the concepts of “less than” and “equal to” to form a relationship of “less than or equal to.” This explanation is useful for basic numerical comparisons, which indicate when one number is less than or equal to another.

8. Shifting perspective: Complexity in interval mathematics

However, as we enter the realm of mathematical intervals, the duality of the symbol “ \leq ” becomes deeper. Consider the interval “ $(-\infty, 15] = \{x \in \mathbb{R} \mid x \leq 15\}$ ”, where “ x ” is a real number. At first glance, this interval seems consistent with the traditional explanation that “ x ” is less than or equal to 15. This interval is actually the union of two parts: all real numbers less than 15, and the specific value 15 itself. In this case, the symbol “ \leq ” becomes the expression “less than or equal to”. This transformation leads us to a deeper understanding of the properties of the interval. Intervals not only contain strict inequalities, but can also represent equality at specific points. The symbol “ \leq ” has more meaning here ^[3].

9. The mystery of interval mathematics: the diversity of symbols “ \leq ”

Looking further into interval mathematics, we find that the symbol “ \leq ” shows diversity in this field. It no longer just means “less than or equal to”, but highlights duality in interval mathematics. The boundary of the interval is not only the critical point, it also includes. This duality plays an important role in mathematics, providing us with more precise tools for describing mathematical phenomena and problems.

10. Interpretation in context: Flexibility of the symbol “ \leq ”

As can be seen from the above, the symbol “ \leq ” has different interpretations in different situations, though its purpose is to keep the same logical conformity. In standard numerical comparison, it is the embodiment of the “less than or equal to” relationship and is used to determine the size relationship of numerical values. In the field of mathematical intervals, however, it has a much richer meaning, expressing duality and the ability to include specific points.

Therefore, when we use the symbol “ \leq ,” we need to interpret its meaning in a specific context. This flexibility allows us to communicate mathematical concepts more accurately and to better understand the application of mathematics in different fields. By delving into the dual meanings of the symbol “ \leq ,” we are able to explore the deeper mysteries of the mathematical world.

11. Reveal the duality of the symbol “ \leq ”

The symbol “ \leq ” has a dual identity in its nature, which ADAPTS and takes on different meanings in different contexts. This duality reflects the complex interplay of mathematical relationships. Depending on the context, this symbol can mean “less than or equal to” or “less than and equal to.”

12. Standard inequalities and traditional bounds

In the standard inequality, the symbol “ \leq ” follows the familiar definition of the relation “less than or equal to”. When we see $a \leq b$, we understand that a is either less than or equal to b . This explanation applies to our daily numerical comparisons and helps us compare the sizes of different quantities.

13. Transcendence in interval mathematics - revealing the dual meanings of the symbol “ \leq ”

In the field of mathematics, symbols and relations between symbols often carry rich mathematical concepts

on the purpose of keeping the same logical conformity. While in interval mathematics, the meaning of the symbol “ \leq ” goes beyond the traditional definition, showing a diverse connotation. This symbol is usually understood as “less than or equal to”; however, it means much more in interval mathematics. In fact, this symbol also represents a special kind of inclusive relationship in interval mathematics, whose meaning depends on the particular context and situation ^[4].

Consider the interval “ $(-\infty, 10]$ ”, which includes the one endpoint 10. In traditional mathematics, the symbol “ \leq ” represents the less than or equal relationship; that is, any number less than or equal to 10 can be classified into this range. In interval mathematics, however, the relationship becomes richer. In this particular context, the symbol “ \leq ” not only represents a numerical size relationship, but also represents a kind of inclusiveness between mathematical objects. The boundary values $-\infty$ and 10 in the interval “ $(-\infty, 10]$ ” satisfy both the less than or equal to relation. Thus, the symbol “ \leq ” here conveys a relationship of “less than and equal to”, emphasizing the special status of the boundary values of the interval.

Therefore, the symbol “ \leq ” is not a fixed mathematical concept, and its interpretation needs to be carried out according to the specific situation. The same symbol may take on different meanings in different mathematical contexts. This leads to an important principle: context dictates interpretation. When we use the symbol “ \leq ”, we must make clear the exact relationship it represents in context.

From a higher-level perspective, we can reveal the duality of the symbol “ \leq ” and further enrich our understanding of it. This comprehensive perspective requires us to go beyond the traditional mathematical boundaries and expand the cognition of mathematical symbols. The symbol “ \leq ” represents the size relationship in traditional mathematics, while it is more inclusive in interval mathematics. This duality prompts us to realize that mathematical symbols may not exist independently, and that they may acquire different interpretations and meanings in different mathematical systems and concepts.

14. The evolution and mathematical background of symbols

The symbol “ \leq ” has evolved in different mathematical contexts, giving rise to different interpretations. In one case, it represents the “less than or equal to” relationship. This interpretation is very useful in standard numerical comparisons, helping us to determine the size relationship between the numbers. In interval mathematics, however, it takes on a different meaning of “less than and equal to.”

15. Thought-provoking: Possibility of multiple interpretations of mathematical symbols

In summary, the duality of symbol “ \leq ” should trigger our thinking about mathematical symbols. It may not be static, but may be changing and evolving according to the context. We need to interpret its meaning in light of a specific mathematical context in order to accurately convey mathematical relationships and concepts. By understanding the dual interpretations of the symbol “ \leq ,” we can explore the mysteries of mathematics more deeply, better apply it to solve practical problems, and pursue a higher level of understanding in the field of mathematics^[5].

16. Conclusion: Harmonious duality of symbol “ \leq ”

In the symphony of mathematical symbols, the symbol “ \leq ” represents a harmonious duality. It is used flexibly in different mathematical contexts, resulting in dual interpretations. While often repetitively interpreted as “less than or equal to,” it translates elegantly into the deeper meaning of “less than and equal to” when exploring the complex world of interval mathematics. This depth of exploration encourages us to appreciate the elegant beauty of mathematical notation while simultaneously embracing its multifaceted nature.

17. Symbolic variations: From tradition to interval mathematics

In traditional numerical comparisons, the symbol “ \leq ” is

often interpreted as a relation of “less than or equal to.” This definition plays an important role in the field of mathematics, helping us compare the sizes of different quantities. However, in entering the complex world of interval mathematics, we find another manifestation of this symbol.

18. An elegant turn in interval mathematics

In interval mathematics, the symbol “ \leq ” elegantly transforms its meaning. It is no longer limited to “less than or equal to”, but further implies “less than and equal to” properties. This shift is manifested at the endpoints of the interval, which are not just critical values, but also part of the interval. This change emphasizes the adaptability and diversity of mathematical symbols in different fields.

19. Embrace versatility and depth of understanding

An in-depth exploration of the dual meanings of the symbol “ \leq ” encourages us to embrace the multifaceted nature of mathematical symbols. By understanding variations of the symbol “ \leq ” in different contexts, we can develop a deeper understanding of mathematical concepts. Due to limited length of this article, though it intends to focus on the analysis of the two different definitions of the symbol “ \leq ” only, by the same token, the two different definitions of the symbol “ \geq ” can also be easily analyzed and concluded.

20. Build a deeper connection between mathematics and the real world

This deeper exploration will also strengthen our connection to the language of mathematics. By being exposed to its applications in different fields, we can connect mathematics with the real world to better solve problems: linguistically as it is no longer appropriate for all English speakers to say “less than or equal to” to express the symbol “ \leq ” as this expression is concluded herein insufficient and inaccurate, “not greater than” or other proper expressions may be used instead; and in similarity “not less than” or other proper expressions may be used to express the symbol “ \geq ” instead of the current

“greater than or equal to.”

21. The symbol “ \leq ” : part of the beauty of mathematics

Taken together, the symbol “ \leq ” plays a colorful role in mathematics. It can not only be the embodiment of “less

than or equal to,” but also show the diverse connotation of “less than and equal to” in interval mathematics. By exploring different interpretations of this symbol, we can not only better understand mathematics, appreciate the diversity and depth of its beauty, but also build a deeper connection between mathematics and the real world, such as the linguistic contribution.

Disclosure statement

The author declares no conflict of interest.

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